

2.2-POSTULATES OF QUANTUM MECHANICS

Classical mechanics-Energy is **continuous**

Quantum Mechanics- Energy is **discontinuous, quantized (fixed)**.

1. POSTULATES OF QUANTUM MECHANICS

(i) Postulate-1 (*Wave functions*)

Every system can be described by a well behaved wave function, Ψ

- *Properties of well-behaved wave functions* – continuous, single valued, disappear at extreme limit, normalized and orthogonal.
- Condition for normalization : $\int \Psi_i^* \Psi_i \, d\tau = 1$
- Condition for orthogonality : $\int \Psi_i^* \Psi_j \, d\tau = 0$

(ii) Postulate-2 (*Operators*)

- Each observable has a characteristic operator.
- *Concept of operators* – mathematical symbol.

Linear operators: (Examples: Differential and integral operators)

Non-linear operators : (*Examples:* logarithmic; Square root)

Commutative (Commuting operators) ($AB\Psi = BA\Psi$): Position and momentum operators do not commute (*Uncertainty principle*)

$$\text{3D Laplacian operator, } \nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$$

$$\text{Hence, 1D Laplacian operator is } \frac{d^2}{dx^2} \text{ or } \frac{d^2}{dy^2} \text{ or } \frac{d^2}{dz^2}$$

$$\text{A 2D Laplacian operator in x and y is } \nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}$$

Direction of operation-operating from left to right.

- *Total energy* operator (Hamiltonian operator), $H : (-\hbar^2 / 8 \pi^2 m) \nabla^2 + V$

(iii) Postulate-3 (Schrodinger Equation)

All systems satisfy an equation of the form $H\Psi = E\Psi$ called *Schrodinger Equation*

(iv) Postulate-4 (Eigen equation)

The observable of a system is given by the *Eigen equation*: $A\psi = a\psi$; Where , A is *Eigen operator*, ψ is *Eigen function*. and a is *Eigen value*.

NB: Schrodinger Equation is a special form of the Eigen equation, $A\psi = a\psi$

NB: (i) ae^{ax} will be an Eigen function for $\frac{d^n}{dx^n}$

(ii) "Asinax & Acosax" will be Eigen functions for $\frac{d^2}{dx^2}$

(iii) "Asinax & Acosax" will not be Eigen functions for $\frac{d^n}{dx^n}$ if n is odd

(v) Postulate-5 (Average value or Expectation value)

The average or expectation value is given by the integral

$$\langle E \rangle = \frac{\int \psi^* H \psi \, d\tau}{\int \psi^* \psi \, d\tau}$$

Where ψ^* is the complex conjugate of ψ