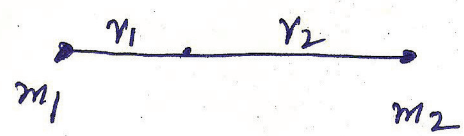
No change in inter nuclear distance, Hence, $V = 0$
There are two angular variables viz., θ & ϕ



$$m_1 r_1 = m_2 r_2$$

$$r_1 + r_2 = R$$

$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 (r_1\omega)^2 + \frac{1}{2} m_2 (r_2\omega)^2$$

$$= \left(\frac{1}{2} m_1 r_1^2 + \frac{1}{2} m_2 r_2^2\right)\omega^2 = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2)\omega^2$$

$$= \frac{1}{2} I\omega^2$$ "|||^r $\frac{1}{2} mv^2$ for linear motion"

$$= \frac{L^2}{2I}$$ where; $L = I\omega$ "|||^r $p = mv$"

$$\hat{H} = \hat{KE} + \hat{PE} = \hat{KE} = \frac{L^2}{2I}$$

Schrodinger equation $H\psi = E\psi \Rightarrow \frac{L^2}{2I}\psi = E\psi$

L^2 in polar coordinates is

$$L^2 = -\frac{h^2}{4\pi^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

$$-\frac{h^2}{8\pi^2 I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\psi = E\psi$$

i.e., $$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{8\pi^2 I}{h^2}E\psi = 0$$

The equation can be solved by the method of separation of variables. The solution expected is of the form

$$\psi_{\theta,\phi} = \Theta(\theta).\,\Phi(\phi)$$

Substituting for ψ we get,

$$\frac{\Phi}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{\Theta}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}+\frac{8\pi^2I}{h^2}E\,\Theta\,\Phi=0$$

Multiplying both sides by $\frac{\sin^2\theta}{\Theta\,\Phi}$ we get

$$\frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2}+\frac{8\pi^2I}{h^2}\sin^2\theta\,E=0$$

$$\frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{8\pi^2IE}{h^2}\sin^2\theta=-\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2}$$

$$= m^2 \text{ (say)}$$

$$\therefore \frac{\partial^2\Phi}{\partial\phi^2}+m^2\Phi=0 \quad \text{------- (1)}$$

$$\frac{\sin\theta}{\Theta}\left(\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{8\pi^2IE}{h^2}\sin^2\theta-m^2=0 \quad \text{---- (2)}$$

The solution for equation-2 is

$$\Phi = e^{\pm im\phi} \quad \text{for} \quad 0 \le \phi \le 2\pi$$

$$m = 0, \pm 1, \pm 2, \dots$$

The normalised function Φ is

$$\Phi = \frac{1}{\sqrt{2\pi}} e^{\pm im\phi}$$

Equation-2 can be furthermodified as follows:

Multiply by $\left(\frac{\Theta}{\sin^2\theta}\right)$ both sides.

$$\frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\Theta}{\partial\theta}\right) + \left(\frac{8\pi^2 IE}{h^2} - \frac{m^2}{\sin^2\theta}\right)\Theta = 0$$

$$\frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\Theta}{\partial\theta}\right) + \left(\beta - \frac{m^2}{\sin^2\theta}\right)\Theta = 0 \quad \text{--- 3}$$

Equation-3 is similar to "associated Legendre equation" with

$$\beta = l(l+1)$$

$\therefore$ Equation-4 has its solution of the form

$$\Theta = P_l^m(x) = (1-x^2)^{m/2} \cdot \frac{d^m}{dx^m} P_l(x)$$

"Associated Legendre polynomial" of degree-l

$l \geq m$ & $m = 0, 1, 2, 3, \dots$

where,

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l}(1-x^2)^l \quad ; \text{ where } x = \cos\theta$$

"Legendre polynomial" of degree-l

Moreover

$$\beta = l(l+1) = \frac{8\pi^2 I E}{h^2}$$

$$\therefore E = \frac{h^2}{8\pi^2 I} l(l+1)$$ "Rotational energy" quantised by l.

Hence,

$N P_l^m(\cos\theta) . N' e^{\pm im\phi}$

$$\psi(\theta,\phi) = \Theta\,\Phi = \left[\frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m(\cos\theta)\frac{1}{\sqrt{2\pi}}e^{\pm im\phi}$$

"Spherical harmonics."

Where N & N' are the normalisation constant as given on RHS.

Allowed values

l =	0	1	1	2	2	2	3	3	3	3
m =	0	0	1	0	1	2	0	1	2	3

NB: $l \geq m$

$$P_0^0(x) = P_0^0(\cos x) = P_0(\cos x) = \frac{1}{\sqrt{2}}$$

$$P_1^0(x) = \left(\tfrac{3}{2}\right)^{1/2} P_1(\cos x) = \left(\tfrac{3}{2}\right)^{1/2}\frac{1}{2^1 1!}\frac{d}{dx}(1-x^2)$$

$$= \left(\frac{3}{2}\right)^{1/2}\frac{1}{2}(-2x) = -\left(\frac{3}{2}\right)^{1/2}\cos x$$